Complex-valued Deng Entropy

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Abstract

Complex evidence theory has been applied to several fields due to its advantages in modeling and processing uncertain information. However,to measure the uncertainty of the complex mass function is still an open issue. The main contribution of this paper is to propose a complex-valued Deng entropy. The complex-valued Deng entropy can effectively measure the uncertainty of the mass function in the complex-valued framework. Meanwhile, the complex-valued Deng entropy is a generalization of the Deng entropy and Shannon entropy. That is, the complex-valued Deng entropy can degenerate to classical Deng entropy when the complex-valued mass function degenerates to a mass function in real space. In addition, the proposed complex-valued Deng entropy can also degenerates to Shannon entropy when the complex-valued mass function degenerates to a probability distribution in real space. Some numerical examples demonstrate the compatibility and effectiveness of the complex-valued Deng entropy.

Keywords: complex-valued mass function, Deng entropy, complex-valued Deng entropy, Shannon entropy, uncertain information

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1. Introduction

How to model and process uncertain information has received increasing attention, and a number of theories have been developed, such as probability theory[1], fuzzy set [2], intuitionistic fuzzy set[3], pythagorean fuzzy set[4], and rough sets[5]. Among these theories, probability theory is fundamental to most theoretical studies and practical applications[6, 7, 8, 9]. Faced with complex problems, probability is extended from real to complex spaces. For example, researchers introduced complex-valued probability into quantum mechanics and quantum game theory to describe interference phenomena[10, 11].

Moreover, as another extension of probability theory, the Dempster-Shafer evidence theory[12, 13], it also has own advantages, the support degree of propositions in evidence theory is not point estimation but interval estimation, and the support degree of correct propositions is further improved after Dempater combination rule. These advantages lead to the wide application of evidence theory in the fields of decision making[14, 15], risk analysis[16], pattern recognition[17], target clustering[18] and other hybrid[19, 20].

Recently, Pan and Deng proposed a complex-valued evidence theory by introducing the phase angle into the mass function, and defined the Dempater combination rule in the complex-valued framework[21]. In the framework of complex-valued evidence theory, it is experimentally demonstrated based on the data set that the complex-valued mass function is more reasonable for modeling uncertain information. This work promotes the application of complex-valued evidence theory in engineering, however there is no mention of uncertainty measurements of complex-valued mass functions in complex-valued evidence theory. Therefore, to reasonably measure the uncertainty of the complex-valued mass function is still an open issue.

In this paper, we propose a generalized Deng entropy, named as complex-valued Deng entropy, which fully considers the effect of phase angle in support

and can effectively measure the uncertainty for the complex-valued mass func-

tion. Concurrently, it is compatible with classical Deng entropy and Shannon

entropy, i.e, when the complex-valued mass function degenerates to a real-

valued mass function, the complex-valued Deng entropy degenerates to clas-

sical Deng entropy or Shannon entropy. Some numerical examples explain the

compatibility and effectiveness of the complex-valued Deng entropy.

The remaining work is organized as follows. Section 2 introduces the knowl-

edge of the complex-valued mass function and Deng entropy. Complex-valued

Deng entropy is presented in section 3. In section 4, some numerical examples

to explain the compatibility and effectiveness of complex-valued Deng entropy

are presented. Section 5 summarizes the work.

2. Preliminaries

In the section, The related concepts of complex-valued mass function and

Deng entropy are introduced.

2.1. Complex-valued mass function

In this subsection, we introduce the concept of complex-valued mass func-

tion and mass function, and the relationship between them.

Definition II.1 (Frame of discernment)[12, 13]

Suppose there exists a positive definite, non-negative, mutually exclusive set

$\mathbb{E} = \{E_1, E_2, \cdots, E_j, \cdots, E_n\}$, denoted as frame of discernment, whose power

set is described as follows:

$$2^{\mathbb{E}} = \{\varnothing, \{E_1\}, \{E_2\}, \cdots, \{E_n\}, \{E_1, E_2\}, \cdots, \mathbb{E}\} \tag{1}$$

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Definition II.2 (Complex-valued mass function)[21]

There exists a mapping function from $2^{\mathbb{E}}$ to $[0,1]$ that

$$|\mathbb{CM}| : 2^{\mathbb{E}} \to [0,1] \tag{2}$$

where $|\mathbb{CM}|$ is modulo of the complex-valued $\mathbb{CM}$. $\mathbb{CM}$ satisfies that

$$\mathbb{CM}\left(\varnothing\right) = 0, \tag{3}$$

$$\mathbb{CM}\left(A\right) = \mathbb{M}\left(A\right) e^{i\pi\theta_A} = x_A + i y_A \tag{4}$$

$$\sum_{A \subseteq \mathbb{K}} \mathbb{M}\left(A\right) = \sum_{A \subseteq \mathbb{K}} \sqrt{x_A^2 + y_A^2} = 1 \tag{5}$$

$\mathbb{CM}\left(A\right)$ is the form of Euler's formula, $\mathbb{CM}\left(A\right)$ is the support degree for the proposition, named as complex-valued mass function, and $\pi\theta_A$ is the phase angle, satisfying the $\pi\theta_A \in [-\pi, \pi]$, that is, $\theta_A \in [-1, 1]$.

Definition II.3 (Mass function)[12, 13]

For a map $\mathbb{M}$ from $2^{\mathbb{E}}$ to $[0,1]$, that is, $|\mathbb{M}| : 2^{\mathbb{E}} \to [0,1]$, and satisfies that

$$\mathbb{M}\left(\varnothing\right) = 0, \sum_{A \subseteq \mathbb{K}} \mathbb{M}\left(A\right) = 1 \tag{6}$$

$\mathbb{M}\left(A\right)$ is denoted as mass function, which indicates the support degree of the evidence for proposition A. Obviously, $\mathbb{M}\left(A\right)$ is a special case of $\mathbb{CM}\left(A\right)$, i.e., when $\theta_A = 0$, $\mathbb{CM}\left(A\right)$ degenerates to a mass function $\mathbb{CM}\left(A\right)$ of real-valued space.

2.2. Deng entropy

Benefit from the application of Shannon entropy in uncertainty measurement under a probability distribution framework, several researchers have

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proposed uncertainty measures for real-valued mass functions in evidence theory, such as Dubois and Prade's weighted Hartley entropy[22], Hohle's confusion measure[23], Yager's dissonance measure[24], Deng entropy[25] and other[26, 27, 28]. These entropies can effectively measure uncertain information and thus are widely studied and applied in various fields. Taking Deng entropy as an example, Abelln analyzed the five properties of Deng entropy[29], Kang et al. analyzed the distribution of maximum Deng entropy[30]. Liu et al. established the relationship between Deng entropy and other non-additive entropies[31]. Moral-García et al. critiqued the Deng entropy and proposed maximum of entropy for belief intervals[32, 33]. Deng proposed information volume of mass function based on fractal idea based on Deng entropy[34]. Buono and Longobardi proposed the dual entropy of Deng entropy[35]. Pan et al. introduced the idea of power-law fractal of maximum Deng entropy in the correlation coefficient to solve the conflict problem between evidences[36]. Liao et al. introduced Deng entropy in the decision-making process to the selection of effective drugs for the COVID-19[37]. Boulkaboul and Boulkaboul apply Deng entropy to IoT decision-making[38]. Zhou et al. proposed the multiple attribute decision-making method based on Deng entropy[39]. Kazemi et al analyzes the fractional law of Deng entropy and discusses its application[40]. Thus, it is defined as follows.

Definition II.4 (Deng entropy)[25]

Consider a frame of discernment $\mathbb{E}$, $\mathbb{M}$ is a mass function, whose uncertainty measure is described as follows:

$$E_d = \mathbb{M}(A) \, ln \frac{\mathbb{M}(A)}{2^{|A|} - 1} \tag{7}$$

where $|A|$ is the cardinality of A. The Deng entropy is an extension of the Shannon entropy, and when $|A| = 1$, the Deng entropy degenerates to the Shannon entropy. The form of the Shannon entropy is described as follows.

Definition II.5 (Shannon entropy)[41]

For a probability distribution P in the frame of discernment $\mathbb{E}$, its uncertainty measure is as follows.

$$E_d = p(A) \, ln \, p(A) \tag{8}$$

where $p(A)$ is the probability of the event A. The $p(A)$ is a special case of $\mathbb{M}(A)$, when when $|A| = 1$, $\mathbb{M}(A)$ and $p(A)$ are equivalent.

3. New entropy for complex-valued mass function

The methods in Subsection 2.2 have their own advantages in measuring the uncertainty of the mass function. However, these works cannot effectively measure the uncertainty for the complex-valued mass function, therefore, it is still an open issue. In this work, a complex-valued uncertainty measure, called as complex-valued Deng entropy is presented as follows:

Definition III.1 (complex-valued Deng entropy)

$$CE_d = \left\| - \left\| \mathbb{M}(A) \, e^{i\pi\theta_A} \right\| \, ln \frac{\mathbb{M}(A) \, e^{i\pi\theta_A}}{2^{|A|} - 1} \right\| \tag{9}$$

where $\left\| \mathbb{M}(A) \, e^{i\pi\theta_A} \right\|$ is the mold of $\mathbb{M}(A) \, e^{i\pi\theta_A}$. $|A|$ is the number of elements in the proposition A, also noted as the cardinality of A. Further, the complex-valued Deng entropy is reexpressed.

$$CE_d = \left\| - \left(\left\| \mathbb{M}(A) \, e^{i\pi\theta_A} \right\| \, ln \mathbb{M}(A) + \left\| \mathbb{M}(A) \, e^{i\pi\theta_A} \right\| \, ln e^{i\pi\theta_A} - \left\| \mathbb{M}(A) \, e^{i\pi\theta_A} \right\| \, ln 2^{|A|} - 1 \right) \right\| \tag{10}$$

next, we organize the complex-valued Deng entropy:

$$CE_d = \left\| - \left(\mathbb{M}(A) \, ln \mathbb{M}(A) + i \mathbb{M}(A) \, \pi\theta_A - \mathbb{M}(A) \, ln 2^{|A|} - 1 \right) \right\| \tag{11}$$

Obviously, the complex-valued Deng entropy is very similar to the classical Deng entropy and Shannon entropy, and the relationship between them is

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described as follows:

(i) **When $\theta_A = 0$, the complex-valued Deng entropy degenerates to the classical Deng entropy, i.e.,**

$$CE_d = - \left(\mathbb{M}(A) \, ln\mathbb{M}(A) - \mathbb{M}(A) \, ln2^{|A|} - 1 \right) \tag{12}$$

(ii) **For arbitrary $|A| = 0$, the complex-valued Deng entropy degenerates to the complex-valued Shannon entropy:**

$$CE_d = CH = \| - (\mathbb{M}(A) \, ln\mathbb{M}(A) + i\mathbb{M}(A) \, \pi\theta_A) \| \tag{13}$$

where CH represents complex-valued Shannon entropy.

(iii) **For arbitrary $|A| = 0$ and $\theta_A = 0$, the complex-valued Deng entropy degenerates to the Shannon entropy:**

$$CE_d = -\mathbb{M}(A) \, ln\mathbb{M}(A) \tag{14}$$

4. Numerical examples

In this section, some examples are designed to explain the compatibility and effectiveness of the complex-valued Deng entropy.

Example 3.1 Suppose there is a complex valued mass function $\mathbb{CM}(E_1) = 1$, whose uncertainty information is measured as follows:

$H = -1 \times ln1 = 0$

$CH = -1 \times ln1 = 0$

$E_d = -1 \times ln\frac{1}{2^{|1|}-1} = 0$

$CE_d = -1 \times ln\frac{1}{2^{|1|}-1} = 0$

This example shows that for a perfectly certain proposition, the uncertainty measure is 0 based on H, CH, E_d and CE_d, which is intuitive.

Example 3.2 Exists a frame of discernment $\mathbb{E} = \{E_1, E_2, E_3, E_4\}$, $\mathbb{CM}(E_1) =$

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$\mathbb{CM}(E_2) = \mathbb{CM}(E_2) = \mathbb{CM}(E_4) = \frac{1}{4}$ is a complex-valued mass function, and its uncertainty information is measured as follows.

$H = -\frac{1}{4} \times ln\frac{1}{4} - \frac{1}{4} \times ln\frac{1}{4} - \frac{1}{4} \times ln\frac{1}{4} - \frac{1}{4} \times ln\frac{1}{4} = 1.3863$

$CH = -\frac{1}{4} \times ln\frac{1}{4} - \frac{1}{4} \times ln\frac{1}{4} - \frac{1}{4} \times ln\frac{1}{4} - \frac{1}{4} \times ln\frac{1}{4} = 1.3863$

$E_d = -\frac{1}{4} \times ln\frac{\frac{1}{4}}{2^{|1|}-1} - \frac{1}{4} \times ln\frac{\frac{1}{4}}{2^{|1|}-1} - \frac{1}{4} \times ln\frac{\frac{1}{4}}{2^{|1|}-1} - \frac{1}{4} \times ln\frac{\frac{1}{4}}{2^{|1|}-1} = 1.3863$

$CE_d = -\frac{1}{4} \times ln\frac{\frac{1}{4}}{2^{|1|}-1} - \frac{1}{4} \times ln\frac{\frac{1}{4}}{2^{|1|}-1} - \frac{1}{4} \times ln\frac{\frac{1}{4}}{2^{|1|}-1} - \frac{1}{4} \times ln\frac{\frac{1}{4}}{2^{|1|}-1} = 1.3863$

Example 3.3 There is a complex-valued mass function $\mathbb{CM}(E_1) = \mathbb{CM}(E_2) = \mathbb{CM}(E_2) = \mathbb{CM}(E_4) = \frac{1}{4}e^{i\frac{1}{3}\pi}$ in the frame of discernment $\mathbb{E}$, then

$CH = \left\| -\left(4 \times \frac{1}{4} \times ln\frac{1}{4} + i \times 4 \times \pi \times \frac{1}{3}\right) \right\| = 3.1416$

$CE_d = \left\| -\left(4 \times \frac{1}{4} \times ln\frac{\frac{1}{4}}{2^{|1|}-1} + i \times 4 \times \pi \times \frac{1}{3}\right) \right\| = 3.1416$

Example 3.2 and **Example 3.3** are the measures of inaccurate information in real-valued space and complex-valued space, respectively. It is reasonable to see from the results that adding the phase angle leads to an increase of the uncertain information in the complex-valued framework. This also shows that the conditions for the application of CH and H.

Example 3.4 Consider a frame of discernment $\mathbb{E} = \{E_1, E_2, E_3, E_4\}$, for a complex-valued mass function $\mathbb{CM}(\mathbb{E}) = 1$, which has the following uncertainty:

$H = -1 \times ln1 = 0$

$CH = -1 \times ln1 = 0$

$E_d = -1 \times ln\frac{1}{2^{|4|}-1} = 2.7081$

$CE_d = -1 \times ln\frac{1}{2^{|4|}-1} = 2.7081$

In contrast to $\mathbb{CM}(E_1) = 1$, $\mathbb{CM}(\mathbb{E}) = 1$ indicates that for propositions that are completely unknown, in **Example 3.1**, $H = 0$ and $CH = 0$. In **Example 3.4**, it is still $H = 0$ and $CH = 0$. Obviously, in **Example 3.4**, the results for H and CH are not reasonable, indicating that H and CH are more applicable to probability distributions than to mass functions. For E_d and CE_d, the measures in both examples are intuitive, which indicate that E_d and CE_d can be effective measures of uncertainty in mass functions, and also, that E_d and CE_d are com-

patible with H and CH.

Example 3.5 Suppose exists a complex-valued mass function $\mathbb{CM}(\mathbb{E}) = e^{i\frac{1}{2}\pi}$
in $\mathbb{E} = \{E_1, E_2, E_3, E_4\}$, then

$$CH = \left\| - \left(1 \times ln1 + i \times \pi \times \frac{1}{2}\right) \right\| = 1.5708$$

$$CE_d = \left\| - \left(1 \times ln\frac{1}{2^{|4|}-1} + i \times \pi \times \frac{1}{2}\right) \right\| = 3.9388$$

The $CE_d = 2.7081$ and $CE_d = 3.1306$ are the results of uncertainty information measurements based on complex-valued Deng entropy in the real-valued and complex-valued framework, respectively, and it is obvious that $2.7081 < 3.1306$, which is due to the uncertainty information of adding phase angles in **Example 3.5**, this result is intuitive and also shows that CE_d is compatible with E_d. Several examples above explain the relationship between CE_d and E_d, CH, H, that is, CE_d is compatible with E_d, CH, H. Next, two examples are designed to explain the effectiveness of CE_d in measuring uncertain information.

Example 3.6 Suppose the existence of a frame of discernment $\mathbb{E}$, there are four complex-valued mass function, described as follows:

$$\mathbb{CM}_1(\mathbb{E}) = 1, \mathbb{CM}_2(\mathbb{E}_j) = \frac{1}{|\mathbb{E}|}, \mathbb{CM}_3(\mathbb{E}) = e^{i\pi}, \mathbb{CM}_4(\mathbb{E}_j) = \frac{1}{|\mathbb{E}|}e^{i\frac{1}{|\mathbb{E}|}\pi}$$

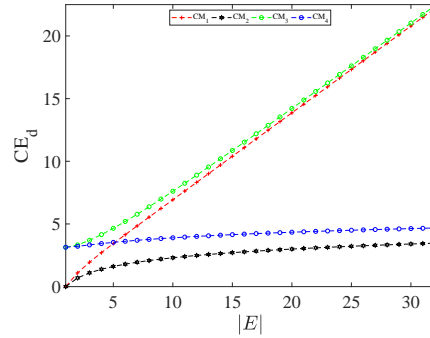| $|E|$	E
1	$\{E_1\}$
2	$\{E_1, E_2\}$
3	$\{E_1, E_2, E_3\}$
4	$\{E_1, E_2, E_3, E_4\}$
5	$\{E_1, E_2, E_3, E_4, E_5\}$
j	$\{E_1, E_2, E_3, \cdots, E_j\}$
31	$\{E_1, E_2, E_3, E_4, \cdots, E_{31}\}$
32	$\{E_1, E_2, E_3 \cdots, E_j, \cdots, E_{32}\}$

Table 1: Variable set E.

Figure 1: Uncertainty measures of four complex-valued mass functions based on CE_d

The form of $\mathbb{E}$ is shown in Table 1. Figure 1 shows the uncertainty infor-

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mation for the four complex-valued mass functions. As can be seen in Figure 1, the uncertainty of $\mathbb{CM}_1$ and $\mathbb{CM}_3$ is larger than that of $\mathbb{CM}_2$ and $\mathbb{CM}_4$. This is a reasonable result because the completely unknown information contains more uncertainty relative to the uncertainty information of the probability distribution. Also, there is a difference between $\mathbb{CM}_1$ and $\mathbb{CM}_3$. For $\mathbb{CM}_3$, the phase angle also contains uncertainty information, so the uncertainty of $\mathbb{CM}_3$ is larger, which is 22.4. Similarly, the uncertainty of $\mathbb{CM}_4$ is larger than that of $\mathbb{CM}_2$. This example explains that the complex-valued Deng entropy can effectively measure the uncertainty for the complex-valued mass function.

Example 3.7 Consider two of the following complex-valued mass functions in $\mathbb{E} = \{e_1, e_2, e_3 \cdots, e_j, \cdots, e_{32}\}$:

$\mathbb{CM}_1(E_5) = 0.05, \mathbb{CM}_1(E_4, E_6, E_7) = 0.1,$

$\mathbb{CM}_1(X) = 0.7, \mathbb{CM}_1(\mathbb{E}) = 0.15.$

$\mathbb{CM}_2(E_5) = 0.05e^{i0.2\pi}, \mathbb{CM}_2(E_4, E_6, E_7) = 0.1e^{i0.4\pi},$

$\mathbb{CM}_2(X) = 0.7e^{i0.7\pi}, \mathbb{CM}_2(\mathbb{E}) = 0.15e^{i0.6\pi}.$

Table 2 lists some of the complex-valued Deng entropy when X varies, and more information is in Figure 2. It can be seen from Table 2 that the uncertainty of $\mathbb{CM}_1$ and the uncertainty of $\mathbb{CM}_2$ increase as the elements in X increases, and the complex-valued Deng entropy reaches the maximum when X=32. It is worth noting that for each X, the uncertainty of $\mathbb{CM}_2$ is always larger than that of $\mathbb{CM}_1$, which is intuitive. This example also demonstrates the effectiveness of the complex-valued Deng entropy.

5. Conclusion

Most of the methods can effectively measure the uncertainty for the mass function in the real-valued framework, however, to reasonably measure the uncertainty of the complex-valued mass function is still an open issue. Therefore, we propose the generalized Deng entropy, named as complex-valued Deng entropy. The main contribution of the complex-valued Deng entropy

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X	$\mathbb{CM}_1$	$\mathbb{CM}_2$	$\triangle$
$\{E_1\}$	4.4360	4.8575	0.4215
$\{E_1, E_2\}$	5.2050	5.5686	0.3636
$\{E_1, E_2, E_3\}$	5.7981	6.1266	0.3285
$\{E_1, E_2, E_3 \cdots, E_4\}$	6.3316	6.6337	0.3021
$\{E_1, E_2, E_3 \cdots, E_5\}$	6.8398	7.1204	0.2806
$\{E_1, E_2, E_3 \cdots, E_6\}$	7.3362	7.5985	0.2623
$\{E_1, E_2, E_3 \cdots, E_7\}$	7.8269	8.0733	0.2464
$\{E_1, E_2, E_3 \cdots, E_8\}$	8.3149	8.5472	0.2323
$\{E_1, E_2, E_3 \cdots, E_9\}$	8.8014	9.0212	0.2198
$\{E_1, E_2, E_3 \cdots, E_{10}\}$	9.2873	9.4959	0.2086
$\{E_1, E_2, E_3 \cdots, E_{11}\}$	9.7729	9.9713	0.1984
$\{E_1, E_2, E_3 \cdots, E_{12}\}$	10.2582	10.4474	0.1892
$\{E_1, E_2, E_3 \cdots, E_j\}$	x	y	$y - x$
$\{E_1, E_2, E_3 \cdots, E_{31}\}$	19.4773	19.5776	0.1003
$\{E_1, E_2, E_3, \cdots, E_{32}\}$	19.9625	20.0604	0.0979

Table 2: Variable set X.
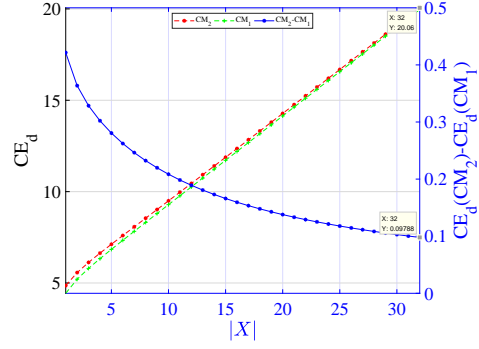
Figure 2: Uncertainty measures of two complex-valued mass functions based on CE_d

is that the uncertainty of the complex-valued mass function can be effectively measured under the framework of complex-valued evidence theory. Also complex-valued Deng entropy has high compatibility. Some numerical examples demonstrate the compatibility and effectiveness of complex-valued Deng entropy. Thus, the new entropy provides a reliable tool to measure uncertain information. Further, we may discuss the properties of the new entropy and the physical meaning in future work.

Declaration of competing interest

The authors have no conflict of interests regarding the publication of this paper.

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